Numerical modelling of tide-induced residual circulation in Sydney Harbour

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Abstract. Tide-induced residual circulation and dispersion in Sydney Harbour has been studied with a vertically integrated, two-dimensional, primitive equation model. The tide-induced residual circulation consists of a series of recirculating gyres that are due to the interaction of the tidal current with the complex coastal geometry and bathymetry of the harbour. Inclusion of the \(S_2\) tide and homogeneous wind stress in the input forcing does not significantly change the residual circulation pattern. Residuals are produced mainly by the \(M_2\) tide and there is no significant nonlinear interaction between the \(M_2\) and the \(S_2\) tide. Tracer simulations show that tidal mixing is limited in the vicinity of the entrance and the flushing rates of different segments of the harbour vary significantly. Lack of similarity between the Eulerian and Lagrangian residual fields demonstrates that the net displacement of material released during a particular phase of the tide has little connection with the mean current observed at fixed locations and is extremely sensitive to the timing and location of release.

Introduction

Sydney Harbour (also known as Port Jackson) is an estuary. Estuaries are important biologically, are highly productive and constitute the prime habitat for many species and the nursery and spawning areas for many more. Commercial fisheries in Sydney Harbour landed 126 ton of prawns, fish and crayfish during 1990–91 (Anon. 1991). But estuaries are often used as a receiving body for urban and industrial waste water. Increased external loading of nutrients usually leads to anoxic conditions, seagrass loss and reduced fishery production. Sydney Harbour has a long history of pollution, high organic loadings and fluctuating oxygen levels with anoxic conditions occurring periodically (Irvine 1980).

The character and health of the estuary depend on the movement of water within the estuary and exchange with the open shelf waters, which, in turn, influences the movement of fish larvae, sediment, introduced chemicals and effluent. This transport of material arises from both advection and mixing by turbulent processes. Estuaries often have complex coastline and bathymetric variations that give rise to strong spatial variations in tidal currents and a marked asymmetry between ebb and flood flows. These lead to effective tidal dispersion and exchange that is many times larger than that resulting from three-dimensional turbulence or vertical shear dispersion (Zimmerman 1986). Quantifying and understanding these mixing processes is therefore of crucial importance for effective management of these regions.

To understand the dispersion and transport process within the harbour, a knowledge of the residual circulation is needed. Residual flows can be produced by local wind stress on the sea surface or driven by lateral density gradients due to non-uniform salinity or temperature distributions. They can also be generated by the tidal flow itself. In the absence of any constant source of freshwater discharge the tide is the dominant cause of water movement in the harbour. Moreover, tidally driven residuals (even those too small to be detected in tidal current meter records) can contribute significantly to the overall long-term transport of water properties because of their persistent features. It was shown by Pingree and Griffiths (1979) that the tide-induced residual stress was consistent with observed sand transport paths in the North Sea. It is also important to determine the tide-induced residual circulation simply because it is a contribution to the mean current measured by current meters. In regions where the mean is dominated by this residual, current meter studies can easily be misinterpreted, particularly if the spatial structure of the residual is not adequately resolved by the current meter array. So this paper studies tide-induced residual circulation, exchange, and dispersion in Sydney Harbour.

Tidal flow in an estuary may be modelled analytically only after making certain assumptions such as the simplified geometry and linear equations. Such a model could be analogous to a model of tides on a sloping shelf (Das and Middleton 1997). Since the nonlinear advective term plays an important role in the tide-induced residual calculation (as will be evident from the results), a single analytical model is usually not adequate.

Though Sydney Harbour forms the nucleus around which Sydney is located, little research work has been...
done on it. Irvine (1980) investigated sediment transport and heavy metal pollution in the harbour. B. V. Hamon et al. (personal communication) compared their numerical results regarding the change in tidal constituents with observations. In their calculations they considered the harbour as a one-dimensional channel of varying width and sectional area, and ignored the Middle Harbour. The present work has been carried out by using the two-dimensional mode of a three-dimensional, sigma co-ordinate, free surface primitive equation model known as the Princeton Ocean Model (POM) developed by Blumberg and Mellor (1987). This model and a model similar to this have been extensively used for modelling tides in bays (Blumberg 1977; Kjerfve et al. 1992) and estuaries (Oey et al. 1985a, 1985b). The model calculates the two-dimensional external mode and a three-dimensional internal mode separately to be numerically efficient. Like most of the numerical simulation of tide and tide-induced residual (Flather and Heaps 1975; Tee 1976; Blumberg 1977; Signell and Butman 1992; Sturley and Bowen 1996) we use only the vertically integrated model (external mode) to study the tidal characteristics of Sydney Harbour. The model does not include all of the estuary zone. Since oceanic plankton of tropical origin can occasionally be traced up to Gladesville bridge (Revelante and Gilmartin 1978), this position is assumed here to be where the oceanic influence vanishes, and the upstream end of the model has been taken to be beyond the bridge. All density variations in the harbour have been neglected and this is a reasonable assumption for Sydney Harbour for most of the year. Moreover, Blumberg (1978) investigated the influence of density variations on estuarine tide and circulations numerically and he found that the discharge through any section, the tidal range, and the tidal phases were largely independent of the density structure. Wind is included in the model to study the effects of wind stress and its changing direction on the tidal recirculation gyres.

‘Many important hydrodynamic processes are convection dominated and they could be better represented in a Lagrangian reference frame. In these situations, the Lagrangian treatments could lead to a better understanding of the underlying physics of the processes’ (Cheng 1988). Therefore, a tracer program has been adapted to the model to compute the movements of water mass in the Lagrangian sense.

The estuary

Sydney Harbour is a drowned river valley on the southeastern coastal plain, has a moderately wide entrance (≈ 3 km) and no significant bar or sill. The bathymetry is complex, with a number of deeps (28 m) separated by shoal water with depths of <3.5 m. A number of large, shallow, muddy bays adjoin the main channel and represent large reservoirs for tidal water. Freshwater input is entirely dependent upon local rainfall runoff, and no permanent rivers or streams enter the system. Although we speak of the Parramatta and the Lane Cove ‘rivers’ these are really only arms of the great estuary forming the harbour, and carry little freshwater flux except in rare flood events.

The Harbour contains West Central South Pacific water carried southwards by the East Australian Current and modified within the estuary by occasional local freshwater runoff and heating and cooling in situ. Ingression of plankton elements of tropical origin, brought southwards by the East Australian Current, could be traced up the estuary to about the Gladesville Bridge (Revelante and Gilmartin 1978). Tidal records prepared by the Maritime Services Board (Hamon et al. 1982) show that there is little change in tidal amplitude and phase from Camp Cove to the head of navigation in Parramatta River. Data collected at different times (Revelante and Gilmartin 1978; Anon. 1992) show that the water column of the estuary is usually well mixed, presumably as a result of low freshwater discharge and tidal turbulence.

Numerical model

The model used here is referred to as the Princeton Ocean Model (POM). A detailed description of the numerical algorithm is given by Blumberg and Mellor (1987).

Basic equations

The governing equations solved by the model for the tidal motion are the shallow-water equations. The model is designed to separate out the vertically integrated equations (external mode) from the vertical structure equations (internal mode). In most of the numerical calculations of tides, vertically integrated shallow-water equations are used. With conversion of three-dimensional horizontal flow field into two dimensions, certain nonlinear terms (similar to Reynolds stress terms) arise. Omission of these terms in the shallow water region is justifiable only if there is negligible vertical structure in the horizontal flow field. As a preliminary test before these terms were neglected, the model was run in internal mode assuming constant density with 10 vertical levels (terrain following σ levels). Our results confirm the Johns and Oguz (1987) conclusion that the depth-averaged and three-dimensional models produce remarkably similar tidal and residual patterns in estuaries. This validates our choice of the two-dimensional model. The governing equations for the depth-averaged two-dimensional model in their transport form are given by

$$\frac{\partial \eta}{\partial t} + \frac{\partial \bar{u} D}{\partial x} + \frac{\partial \bar{v} D}{\partial y} = 0$$

(1)
Tide-induced residual circulation in Sydney Harbour

Fig. 1. Map of Sydney Harbour. Sections over which discharge was computed and measured: S1, from Cannae Point to South Head; S2, from South Head to Obelisk Bay; S3, Cobblers Beach to Grotto Point.

\[
\frac{\partial \bar{u} D}{\partial t} + \frac{\partial \bar{u}^2 D}{\partial x} + \frac{\partial \bar{v} D}{\partial y} - \bar{T}_x - f \bar{v} D \\
+ g D \frac{\partial \eta}{\partial x} = \frac{\tau_x}{\rho_w} - C_D u (u^2 + v^2)^{1/2}
\]

(2)

\[
\frac{\partial \bar{v} D}{\partial t} + \frac{\partial \bar{u} \bar{v} D}{\partial x} + \frac{\partial \bar{v}^2 D}{\partial y} - \bar{T}_y + f \bar{u} D \\
+ g D \frac{\partial \eta}{\partial y} = \frac{\tau_y}{\rho_w} - C_D v (u^2 + v^2)^{1/2}
\]

(3)

where

\[
\bar{u} = \int_{-1}^{0} u \, d\sigma \quad \bar{v} = \int_{-1}^{0} v \, d\sigma
\]

(4)

are the depth-averaged velocity components in the x direction (increasing eastwards) and y direction (increasing northwards) respectively. \( \eta \) is the surface elevation, \( H \) is the depth below the mean sea level, \( D = H + \eta \) is the instantaneous water depth, \( f \) is the constant Coriolis parameter, \( \rho_w \) is the density of the estuarine water (1024 \( \text{kg m}^{-3} \)), and \( F_x \) and \( F_y \) are the horizontal diffusion terms. The horizontal diffusion uses the Smagorinsky formulation (Smagorinsky and Holloway 1965) in which horizontal viscosity coefficients depend on the grid size and velocity gradients. The wind stress components in the x and y directions are \( \tau_x \) and \( \tau_y \) respectively. Bottom stress is parameterized by using a quadratic drag law, and instead of choosing an empirical drag coefficient (Flather and Heaps 1975; Tee 1976; Blumberg 1977) the bottom stress has been chosen to be a function of \( H \) and roughness parameter \( z_0 \) (Blumberg and Mellor 1987). In deriving the above equations it has been assumed that water is incompressible, and that hydrostatic and density variations are negligible. It has been shown by Blumberg (1978) that density variations produce no contribution to the tidal range, discharge and phase.

Grid arrangement and numerical scheme

The region included in the model domain (Fig. 1) has been schematized to fit on a grid 98 by 80 points with grid spacing \( \Delta x = \Delta y = 150 \text{ m} \). Data with 50 m resolution obtained from Maritime Service Board, Sydney, have been used as the source for bathymetry and interpolated linearly on the model grid. The topography has been slightly smoothed to filter subgrid scale features. The bathymetric schematization (Fig. 2) shows the complex nature of the harbour bathymetry. The model uses Cartesian coordinates and an ‘Arakawa C’ grid. Variables \( \eta, H \) and \( D \) are defined at the middle of the grid, whereas \( u \) and \( v \) velocity components are defined at a distance \( \Delta x/2 \)
(or \(\Delta y/2\)) to the east and west (north and south) of \(\eta\).

Equations in finite difference form are stepped forward in time by using a leapfrog time scheme (Blumberg and Mellor 1987). Numerical stability requirements have been satisfied by choosing the computational time step \(\Delta t\) smaller than the critical time step imposed by the Courant–Fredricks–Levy (CFL) criterion,

\[
\Delta t \leq \frac{\sqrt{g D_{\text{max}}}}{\frac{1}{\Delta x} + \frac{1}{\Delta y}}
\]

and a rotational criterion

\[
\Delta t \leq \frac{1}{f}
\]

The stability condition is more stringent, and with grid spacing of 150 m the critical time step is 6 s. In practice, the stability condition is more restrictive and the time step (\(\Delta t\)) chosen for the Sydney Harbour system was 1 s.

**Initial and open boundary conditions**

The initial values of \(\eta, u\) and \(v\) are taken as zero within the model domain. Along the open boundary, the tidal elevation \(\eta\) is prescribed as

\[
\eta = h_0 \sin \omega t
\]

The spectral analysis of tidal records of Sydney Harbour (Hamon et al. 1982) shows a prominent spectral peak near 2 cycles day\(^{-1}\), and also from the tide charts it is evident that \(M_2\) is the dominant semi-diurnal tide in Sydney Harbour (Table 1). So the frequency \(\omega\) has been chosen to be that of the \(M_2\) tide. The tidal records of March 1992 (Anon. 1992) show that the tidal amplitude was almost 1 m, so to compare our model results with the field data collected on 19 March 1992 the analysis has assumed a tidal amplitude of 1 m.

<table>
<thead>
<tr>
<th>Port</th>
<th>Harmonic constants (amplitude in m, phase in degrees)</th>
<th>M2</th>
<th>S2</th>
<th>K1</th>
<th>O1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camp Cove</td>
<td>0.515</td>
<td>0.129</td>
<td>0.148</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>33°50'S, 151°17'E</td>
<td>241</td>
<td>266</td>
<td>121</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>Sydney</td>
<td>0.501</td>
<td>0.126</td>
<td>0.148</td>
<td>0.096</td>
<td></td>
</tr>
<tr>
<td>33°51'S, 151°14'E</td>
<td>237.8</td>
<td>261.3</td>
<td>120.1</td>
<td>80.0</td>
<td></td>
</tr>
</tbody>
</table>

Numerical calculations require longitudinal \((u)\) and lateral \((v)\) velocity to be defined at the open boundary for calculation of advective fluxes. At the open boundary it can be assumed that the volume transport is perpendicular to the open boundary. Zero-gradient boundary condition has been used for the longitudinal velocity. This requirement is met by setting the longitudinal velocity at the boundary to be equal to the adjacent velocity within the model grid. For closed boundaries POM uses half-slip boundary condition, that is, near the boundary only the velocity normal to the boundary goes to zero.

**Results**

The model reached steady state almost within one tidal cycle. There were no significant changes of the tidal motion and residual current from the 2nd to the 3rd cycle. Also, the mean kinetic energy showed no significant change with time. This agrees well with the comment of Bode and Sobey (1984) that, for harmonic forcing in shallow water, initial transients will be dissipated by the bottom friction in one tidal cycle. The present analysis of the solution is based on the 3rd cycle except in the case of zero friction when it is based on the 8th cycle.

Tidal elevation and current velocity over a tidal cycle at grid points \((87,31)\) and \((25,21)\) marked as A and B in Fig. 2 show that there is little variation in the amplitude and phase of the tidal elevation throughout the estuary (figures not shown). There is almost no difference between the two localities in the times at which high water and low water occur; this is in agreement with the Maritime Service Board observations (Hamon et al. 1982). Current velocities vary considerably in magnitude throughout the estuary but the current phase changes very little. The tide in the Harbour is essentially a standing wave. There is a little change \((\approx 3\text{ cm})\) in the tidal amplitude from the mouth of the harbour to the end of the upstream boundary. Since the harbour is shallow and short (i.e. the basin length is very much less than the quarter wavelength of the tide), water level in the entire basin fluctuates up and down simultaneously (i.e. in phase) during the ‘flood’ cycle and ‘ebb’ cycle.

From the numerically simulated tidal velocity distributions over a tidal cycle at intervals of a quarter tidal period, it is evident that the tide propagates through the deep-water region and is steered by the coastal geometry and complex bathymetry. Strong currents are evident where the width of the harbour is narrow or the depth is shallow. This follows from the continuity condition. A combination of these effects produces stronger currents in the Middle Harbour.

**Comparison of model results with field data**

As a check, the numerical results can be compared with the limited field data available (collected by Sydney Water Board on a particular day in 1992). Figure 4a shows the computed and measured discharge across three sections (Fig. 1) near the Heads, over one \(M_2\) tidal cycle. The total discharge at any time along any cross-section
has been calculated by using the following equation:

\[ Q(t) = \sum_{i=1}^{n} U_i(t) \times A_i(t) \]

where \( n \) is the number of grid points along the cross-section, \( U_i(t) \) is the component of depth-averaged velocity at grid point \( i \), and time \( t \), and \( A_i(t) \) is the area of the cross-section of the grid point \( i \) at time \( t \). Considering the \( M_2 \) tide as the only forcing, the discharges calculated along any section are within 20% of those observed. Agreement between these two results in the absence of forces other than the tide indicates that tide is the major cause of water transport in the harbour.

As no experimental data of the tidal elevation at the entrance are available, no comparison can be made of the sea-level rise at any station with that at the entrance. Therefore an attempt has been made here to compare the difference in water level between two localities within the harbour. Figure 4b shows the simulated and measured (at 1 s intervals) difference in water level between HMAS Penguin and Gladesville Bridge. The computed results are consistent with the raw data, and to some extent verify the model configuration.

**Eulerian residual current**

Before presenting the simulation of residual current it is appropriate to define the two different types of time
averages that have been used here. The residual measured by a current meter is an Eulerian residual velocity. The Eulerian residual velocity \( \vec{u}_E \) is calculated by taking the time average of depth-averaged velocity \( \bar{u} \) over an \( M_2 \) tidal cycle. In the finite difference form this is represented as

\[
\vec{u}_E = \frac{1}{N} \sum_{i=1}^{N} \bar{u}
\]

(6)

where \( N \) is the number of time steps in one tidal cycle.

The Eulerian residual transport velocity, \( \vec{u}_T \), is the residual velocity required to transport, within the mean depth \( H \), the same volume of water as is actually transported past the given position (Robinson 1983), and not only includes tide-induced residual effect but also conserves volume transport. It can be defined by

\[
\vec{u}_T = \frac{1}{N} \frac{H}{H} \sum_{i=1}^{N} (H + \eta) \bar{u}
\]

(7)

There is no difference between the \( \vec{u}_E \) and \( \vec{u}_T \) plots for the Sydney Harbour system (Fig. 5) because \( \eta \ll H \). Residual flow consists of a series of recirculation gyres and their sense of rotation is usually opposite to that of adjacent gyres. Formation of these gyres in the model in the absence of wind stress and density stratification (both of which can also produce residual flows) shows that they are produced by the tidal flow.

In estuaries, residual eddies are formed by two mechanisms: the nonlinear bottom frictional force, and the boundary geometrical effect of the coast. Nonlinear bottom friction produces torque on a sloping bottom and in a shear flow. These torques, when advected nonlinearly, produce residual current. In estuaries, because of the complicated geometry, nonlinear terms in the momentum equations become very large and the average of these nonlinear terms over a tidal period gives rise to residual circulation. The eddies around a headland are commonly known as headland eddies. In the harbour most of the eddies can be classified as headland eddies. It has been argued by Middleton et al. (1993) that the recirculation behind a headland depends on the complicated coastal geometry and bottom topography in the vicinity of the tip of the headland. Denniss and Middleton (1994) investigated the effects of bottom friction in the dissipation of recirculating flows.

Near the entrance of Sydney Harbour, the formation of the clockwise gyre near North Head and an anticlockwise gyre near the entrance of the main channel (near South Head) can be explained from the principles of the generation of basin eddies as explained in Robinson (1983). As the water enters the harbour, a strong inertial force causes the current near the entrance to be greater than that on either side. As a result, clockwise and anticlockwise torques are produced on the right and left side. Similarly, torques of an opposite sense are produced during the ebb flow. Torque produces vorticity in the fluid, which will oscillate tidally as the sense of the torque oscillates. However, because of the gradient in the magnitude of the torque, stronger vorticity will be brought in on the flood than is carried out on the ebb. Therefore, over a tidal cycle, clockwise and anticlockwise gyres will be formed at those places.

Residual circulation is generated whenever there is a net flux of tidal vorticity into a particular region over a tidal cycle (Zimmerman 1981). To generate vorticity a

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**Fig. 4.** (a) Comparison of the observed and computed discharge along three cross-sections near Sydney Heads: +, o, * represents the experimental data along the transects from Cannae Point to South Head (S1), South Head to Obelisk Bay (S2) and Cobbler’s Beach to Grotto Point (S3) respectively; flow convention followed is positive for outgoing tide and negative for incoming tide. (b) Comparison of the simulated and observed water level difference between HMAS Penguin and Gladesville Bridge: + represents the raw data points.
spatially inhomogeneous force is required. It has been shown by Nihoul and Ronday (1975) and Sugimoto (1975) that in the case of a tidal oscillating flow, ‘tidal stress’, which arises from the nonlinear effect of the flow pattern on irregular bottom topography and coastline, provides the necessary force. Signell and Butman (1992) presented a summary of papers that explain different generation mechanism for residual flow.

Fig. 5. (upper) Eulerian Residual Velocity ($u_e$) m s$^{-1}$. (lower) Eulerian Residual Transport Velocity ($u_T$, m s$^{-1}$).

**Dynamical balance of the model**

**Nonlinearity**

Tides may generate residual currents through nonlinear bottom friction and nonlinear advective terms in the momentum equations. To study the role played by nonlinear bottom friction on the tidal residual flows we calculated residual flow in the presence of nonlinear advective terms and a linear bottom friction term ($C_D u$) (Fig. 6a). Comparing Fig. 6a and Fig. 5 it can be seen that the circulation pattern near the entrance of the harbour has been changed; this shows that bottom friction plays an important role near the entrance. This agrees well with the findings of Denniss and Middleton (1994) that outside the vicinity of the tips of headlands and the sides of the coast the bottom friction can act as the controller of the flow. With the linear assumption, the magnitude of the residual current decreases; this is because the linear assumption precludes bottom friction as a means of generating residuals and the term acts only as a dissipative term.

It has been shown by Robinson (1983), Zimmerman (1981) and Tee (1976) that if the vorticity ($\zeta$) is considered to consist of residual vorticity ($\zeta_0$) and oscillating vorticity ($\zeta_1$) then the magnitude of residual vorticity ($\zeta_0$) is a balance between its generation by the advection of tidal vorticity by the oscillating flow ($\vec{u} \cdot \nabla \zeta$) and its dissipation by bottom friction and by turbulent eddy diffusion. In terms of the vorticity balance, since less vorticity is advected and the magnitude of the dissipation increases, the residual vorticity becomes small. Therefore, the magnitude of the residual current decreases.

Next, we calculated the residual circulation after omitting all the nonlinear advective terms (2nd and 3rd terms) in Eqns (2) and (3) that represent the interaction of the tidal wave with the basin geometry (Fig. 6b). Not only does the magnitude of the residual current decrease by an order of magnitude but also the pattern changes completely. This demonstrates the importance of inclusion of nonlinear terms in the tide-induced residual calculation as suggested by Nihoul and Ronday (1975), Sugimoto (1975) and Tee (1976). Since no residuals are produced when linear bottom friction is used and advective terms are omitted (not shown), these non-zero residuals are therefore due to the presence of nonlinear bottom friction term.

**Bathymetry**

Zimmerman (1978b) showed that, in the presence of irregular bottom topography, bottom friction can produce a torque that, when advected non-linearly, produces residual flow. To study the effect of bottom topography on residual flow, the harbour is now assumed to be of constant depth, $H$, of 10 m, where 10 m represents the harbour’s mean depth (Fig. 6c). Comparison with Fig. 5 shows that topography changes the circulation pattern quite significantly near the harbour mouth and also inland of the Sydney Harbour bridge. Near the mouth, setting of $H = 10$ increases the tidal current and therefore the inertial effect. As a result,
the size of the gyres near the North Head and near the entrance of the main channel increases.

Idealized flat-bottom bathymetry increases the size of the recirculation gyres near Bradley’s Head, but the headland eddies are not forming properly in the absence of bottom slopes. This indicates that the torque produced by the bottom friction on a sloping topography is important in the formation of headland eddies. This agrees with the findings of Denniss et al. (1995) that the local bathymetry in the vicinity of the headland determines the size and shape of the eddy. Recirculating gyres inland of the harbour bridge disappear in the absence of topographic variability. In other areas, since the increase of mean sea depth is equivalent to decreasing bottom friction, the magnitude of residual current is slightly higher than when real bathymetry is used. Since the circulation patterns in other parts do not change significantly, these gyres must have been produced by the complex shape of the coastline.

**Bottom friction coefficient**

It is important to determine the influence of both bottom friction and roughness length on the solution. The relation between the bottom friction coefficient $C_D$ and the roughness length $z_o$ has been derived by assuming a constant drag law and a logarithmic boundary layer (Gross and Werner 1994) and is given by

$$ C_D = \left( \frac{1}{k} \ln \frac{z}{z_o} \right)^{-2} $$

**Fig. 6.** Tide-induced residuals (a) when linear bottom friction is used, (b) in the absence of advective terms, (c) when depth is assumed to be constant and (d) in the absence of bottom friction term.
where $\kappa$ is von Kármán’s constant and $z$ depends on the depth and the spacing between the bottom and the adjacent terrain following vertical layer (Blumberg and Mellor 1987). A test increase in the value of $z_o$ from 1 cm to 5 cm decreased the magnitude of the residual current but the pattern remained the same (not shown here); this is as expected, because an increase in the value of $z_o$ increases the bottom friction co-efficient $C_D$. This result can also be explained from the steady-state vorticity equation which states that the net amount of vorticity advected into the region is equal to the dissipation and diffusion of steady vorticity formed by the residual current. An increase in $z_o$ increases the dissipation coefficient. Therefore, steady-state vorticity will be small and this implies that the magnitude of residual current will decrease.

Next, we calculated residual flow without the bottom friction term (Fig. 6d). Comparison of Fig. 6d with Fig. 5 shows that in the absence of bottom friction the magnitude of the residual current increases and the residual circulation gyres intensify, with the change in the residual circulation pattern being significant near the entrance of the main channel and Middle Harbour. This is because bottom friction depends on the local water depth and on the magnitude of the tidal current; where the depth is shallow or the current is strong, bottom friction plays a significant role and vorticity is dissipated by the bottom friction. Therefore, in the absence of bottom friction, the magnitude of the residual current will be larger. If no vorticity is dissipated by bottom friction it has to be diffused through horizontal turbulent eddy diffusion which increases the size of the gyres. Even in the absence of bottom friction, residual eddies are formed in a similar way, which confirms that bottom friction mainly acts as a dissipative term throughout the harbour. This also confirms the statement of Denniss and Middleton (1994) that the formation of recirculating gyres near the headland and the side of the coast does not depend on the value of the bottom friction.

**Open boundary conditions**

The results of numerical models depend critically on the basic equations, the boundary conditions and the grid size used in the model. As no experimental data at the open boundary are available the prescribed tidal elevation introduces uncertainty in the open boundary conditions. How this uncertainty affects the results was tested by changing the tidal amplitude from 1 m to 0.5 m. It was found (not shown here) that this decrease in amplitude did not produce any change in the recirculation pattern. Only the magnitude of the residual current decreased, this being a consequence of smaller advection of vorticity.

**Effects of wind stress**

To study the effects of wind on circulation in the harbour, comparisons were made between residuals induced by (i) the wind, (ii) the tide and (iii) the tide and wind. Since regional winds in the summer blow mostly from the north-west and in the winter from the south-east, the model was forced with a spatially and temporally homogeneous wind stress with wind speed 10 m s$^{-1}$. The wind-induced residuals (Fig. 7 upper panels) are larger than the tide-induced residuals (Fig. 5). Headland eddies near Bradley’s Head are formed only under the tidal forcing. The residual flow in the presence of tide and wind (Fig. 7 lower panels) is similar to that of the tide-induced residuals (Fig. 5). It seems that wind effects are dominated by tidal flow in most areas and only a few additional gyres have formed in North Harbour and Rose Bay as an effect of wind. The gyres that were formed in the absence of wind stress, except those formed after the Gladesville bridge and near the entrance of Middle Harbour, are generally not affected by the wind stress. With a south-east wind an anti-clockwise circulation is produced in Rose Bay and two additional gyres are formed in North Harbour. With a reversal of the wind direction a clockwise gyre is formed in Rose Bay, one of the gyres in North Harbour disappears and the remaining gyre changes direction of rotation.

**Interaction with $S_2$ tide**

The value of the ‘form ratio’ $F = (K_1+O_1)/(M_2+S_2) = 0.38$ for Fort Denison indicates that Sydney Harbour is mainly a semi-diurnal tidal region. Therefore, only the influence of the $S_2$ tide on the $M_2$ residual flows will be considered. To do this, residual flows of three different cases have been considered: due to the $M_2$ tide alone, due to the $S_2$ tide alone, and due to a combination of $M_2$ and $S_2$ tides. The amplitude of each constituent was taken according to the value given in Table 1. In the first and the second case residual flows were determined by averaging over their respective tidal periods which is 12.42 solar hours for $M_2$ and 12.0 solar hours for $S_2$ tides. The averaging period for the combination of $M_2$ and $S_2$ tide was derived from the theory of two waves moving in the same direction whose amplitudes and periods are different. Combination appears as the product of two waves: one, a wave with wave number and frequency between those of the two individual waves comprising the original signal; the other, the ‘envelope’, a wave with much larger wavelength and period. Propagation speed of this wave is called the group velocity, and this is the velocity at which energy is carried by the waves. The period of this wave was taken as the averaging period for the third case, which for the present data is 14.7 days. Figure 8a shows the same residual pattern when the tidal amplitude considered was 1 m (Fig. 5).
mentioned above, change in the amplitude only changes the magnitude of the current. With the observed input, the tide-induced residual currents are much smaller in the case of the $S_2$ than the $M_2$ tide (Fig. 8b). Moreover, the $S_2$ tide does not produce any residual gyres. Since these gyres are formed from the inclusion of the advective terms it confirms Tee's (1976) statement that the main contribution to the advective terms is from the $M_2$ constituent. Although the combination of $M_2$ and $S_2$ tides has been considered (Fig. 8c), the residual flows are dominated by the $M_2$ tide and there is no significant nonlinear interaction between $M_2$ and $S_2$ tide in the Sydney Harbour region.

**Dispersion of pollutants**

Evidence of the existence of residual eddies has been confirmed from hydraulic models (Yanagi 1974; Sugimoto 1975) and also from field current meter measurements (Zimmerman 1976; Tee 1977). Tide-induced residual eddies play a very important role in the dispersion of matter (Yanagi 1974; Sugimoto 1975). According to Zimmerman (1978a), residual eddies contribute to the longitudinal dispersion in two different ways. First, residual eddies produce an irregular distribution of residual current shear which in collaboration with the small-scale (tide-induced) turbulence enhances diffusion (Sugimoto 1975). Secondly, superposition of an oscillatory (tidal)
motion on a Eulerian residual velocity field allows water parcels to escape from one tidal pathway to another and gives rise to a dispersion process in the Lagrangian sense.

A tracer program was adapted to the model in order to track the path of ideal tracers and analyse the dispersion process in the harbour; at each internal time step the tracer is advected and the new position is recorded. In constructing trajectories of water parcels, the displacement vectors are computed from the tidal velocity where the labelled water parcel is located at that instant of time. The local tidal velocity is obtained by means of linear interpolations from known data at the surrounding grid points. If a tracer moves out of the region of interest it is marked as dead and is not further propagated. In this study, tracers at three places in the model were introduced and their paths were tracked for six tidal periods (Fig. 9). Comparing Fig. 9 with Fig. 5 it can be seen that at the beginning the directions of the motion of all tracers agree well with the direction of the residual gyres at those places. But later the motion of the innermost tracer in the adjoining bay and the outermost tracer in the Middle Harbour confirm that the dispersion process in the harbour depends on the interaction of tidal current with the residual velocity field. This confirms the statement by Tartinville et al. (1997) that although Eulerian residual circulations exhibit large-scale ‘horizontal gyres’ this does not mean that Lagrangian particles are likely to follow the streamlines of the horizontal circulations.

According to Zimmerman (1981), where residual eddies constitute the largest scale the residual eddy diffusion coefficient can be expressed as a function of residual velocity and its length scale. But when the paths of the tracers depend both on the tidal flow and also on the residual flow field, as is the case in Sydney Harbour, the diffusion coefficient can not be described by either the tidal parameters or the parameters of the residual current velocity field (Zimmerman 1978a).

**Lagrangian residual current**

‘It has long been recognised that movements of water mass in tidal estuaries are more inclined to be Lagrangian than Eulerian in nature’ (Cheng 1988). The Lagrangian residual velocity of the Sydney Harbour estuary is obtained by dividing the net displacement of a particle during a tidal cycle by the tidal period (Cheng 1988). This can be written as

$$\bar{u}_L = \frac{X(t_o + T) - X(t_o)}{T}$$  \hspace{1cm} (8)

where $T$ is the tidal period and $t_o$ is the initial time.
The particles were released from each grid cell at high/low tide and tracked for one tidal cycle. Figure 10 shows the tide-induced Lagrangian residual current in Sydney Harbour calculated between successive high tides and low tides. It is not surprising to find that the net Lagrangian displacement of the water parcel depends on the time (tidal current phase) when the water parcel is labelled and released; since the net displacements are functions of the bathymetry enclosed within the trajectories, there is no reason to expect that the net displacements should be identical.

The lack of similarity between this field (Fig. 10) and the Eulerian residual velocity (Fig. 5) demonstrates that the net displacement of material released during a particular phase of the tide has little connection with the mean current observed at fixed locations. Regions where the Lagrangian residual vectors have strong spatial variability indicate regions where water parcels will become widely spread. Pingree and Maddock (1977) have pointed out that, except in the special case of a rectilinear tidal flow, the Lagrangian residual velocity is not the same as the Eulerian residual transport velocity.

**Flushing time**

Flushing time is a crude measure of how long, on average, a parcel of water would remain in the harbour before being transported onto the shelf. Shorter flushing time means that introduced substances can be quickly flushed out of the harbour. Longer flushing time may allow introduced substances to accumulate within the harbour. These times have a most important influence on the water quality in the system.

The most predictable mechanism for flushing a small, well mixed tidal estuary is the tidal circulation accompanying the regular rise and fall of water of the astronomical tide, but a number of other factors can also produce flushing, for example the wind and the freshwater discharge. However, these are not as regular or as predictable as the astronomical tide. For this reason, the more conservative estimate of flushing characteristics can be realized by considering the most limited flushing condition, that of the astronomical tide acting alone.
(1971) using field data, the flushing time may be estimated as the ratio of the volume of the region to the volume flux entering or leaving the region. It has been argued by Tartinville et al. (1997) that this method is unlikely to yield an acceptable estimate of the flushing time when the residence time (i.e. the time taken by a particle to leave the region) varies significantly in space. Therefore in estimating the flushing time of Mururoa Atoll lagoon Tartinville et al. averaged the residence time over the volume of the lagoon. Since residence time is often difficult to measure, Tartinville et al. used a numerical model to estimate it.

An estimate of the flushing time scale of Sydney Harbour was also found numerically by using the tracer program mentioned above (see Dispersion of pollutants). Assumption of a well mixed condition inside the domain suggests a single way of evaluating the flushing time (Tartinville et al. 1997). The mass \( m(t) \) of the tracer particles present in the harbour at any time \( t \) is then given by

\[
m(t) = m(0) \exp \left(-\frac{t}{B}\right)
\]

where \( m(0) \) is the initial mass and \( B \) represents the time scale, which might be considered the flushing time scale.

Following the above procedure at the beginning of the numerical simulation, 2048 passive tracers (i.e. one tracer in each numerical grid cell) were released and as the tracer transport module is integrated in time, the total mass of the tracer particles remaining in the Harbour was recorded. The simulation was stopped after 83 days when the rate of particles leaving became very small (i.e. 1% tracer leaves in 30 days) which indicates that tidal exchange becomes less effective. The flushing time is estimated as the time scale providing the best agreement, in the least-square sense, between the exponential law – Eqn (9) – and the time series of the mass left in the Harbour (Fig. 11a). According to this procedure, the flushing time of Sydney Harbour is 225 days.

To investigate how different parts of the harbour flush, the domain was divided into four segments (Fig. 11b) and the percentage of the initial mass of each segment was also recorded for the same period. It is evident from the time series of the percentage of initial mass of four segments (upper panel of Fig. 12) that the assumption of a horizontally well mixed condition inside the domain does not apply for Sydney Harbour. Therefore, the flushing time estimated on this assumption should be regarded as the lowest bound of the value. Time series also shows that all segments except Segment 2 are being flushed. In fact, the mass in Segment 2 exceeds its initial mass because particles have been received from neighbouring sections. Although Segment 4 was flushed more than Segment 3 over this period, flushing is rapid in Segment 3 in the first few days. This indicates that rapid exchange occurs near the entrance of the harbour mouth. This also explains why the time dependency of \( m(t) \) in Fig. 11a departs somewhat from an exponential function during the first 20 days of the simulation.

Fig. 11. Percentage of initial mass of the tracer particles left in the Harbour. (b) The four segments of the harbour domain.

We agree with Signell and Butman’s (1992) comment that a single flushing time for the harbour is a somewhat artificial construct, since material released in the strong tidal channels near the harbour mouth is flushed much more rapidly than for the harbour as a whole, and material released around the periphery of the harbour in weak tidal currents is flushed more slowly. From the practical point of view it is important to know how long an area of harbour takes to effectively flush the introduced materials. Therefore, in the next section we have investigated the relative flushing rate of different segments of the harbour.
Fig. 12. (Upper panel) Percentage of initial mass of the tracer particles left in the four segments of the Harbour domain. (Lower panel) Relative tidal flushing at these segments during a 15-day period. In both cases (a), (b), (c) and (d) represent Segment 1, Segment 2, Segment 3 and Segment 4 respectively.

Relative flushing rate

Knowledge of relative flushing rate of different segments of the harbour helps to locate areas of low flushing that might explain why the water quality is poor in that area. To calculate the relative tidal flushing rates, at the beginning of the numerical simulation a passive tracer was released into all the numerical grid points of a particular segment whose flushing rate was to be determined, and all other segments were kept free of tracers. Tracers were allowed to come back to the original segment even when they went into the other segments, and they were declared dead when they reached the open boundary. Simulation was continued for 15 days and as the simulation proceeded the relative amount of substance (as a percentage of the initial mass) remaining in the segment was calculated. A similar procedure was adopted by Sheng et al. (1996) to calculate the relative tidal flushing rate of Sarasota Bay. The simulation indicated that the flushing through Segment 3 is fastest and through Segment 1 is slowest (Fig. 12 lower panel). This also suggest that the residence time in the vicinity of the entrance of the Harbour is much smaller than anywhere else.

Assuming a well mixed condition inside the domain of a segment, the flushing time of four segments can be estimated from the exponential law in Eqn (9). Here, flushing time means the time taken by the particles to leave the particular segment. They can either move into adjacent segments or leave the harbour. Estimated flushing times are 113 days for Segment 1, 67 days for Segment 2, 23 days for Segment 3, and 42 days for Segment 4. A comparison between the upper and lower panel of Fig. 12 shows that flushing time of Segment 2 and Segment 1 depends on the way the particles are introduced in the harbour. The flushing time of Segment 2 is shorter than for Segment 1 when particles are introduced only in that particular segment. The situation reverses when particles are present all over the harbour.

In this simulation tracers were not allowed to come back after they reached the open boundary. In reality, a portion of particles that have left the harbour during the ebb tide will be back in the next flood cycle. This might increase the flushing time of Segment 3 and Segment 4, so estimated values for Segment 3 and Segment 4 should be regarded as the lower end of the range. This exercise has been done to show how flushing time changes from the mouth of the harbour to its upstream end. These numbers are not absolute because they depend on the phase of the tide when the particles were released and the amplitude of the forcing. In this calculation we have used only a $M_2$ tidal constituent of 0.5 m amplitude. Since we are considering only the tidal effects, these numbers would be at the upper end of the range. Other factors such as wind and freshwater discharge might reduce the flushing time considerably.

Since Segments 3 and 4 occupy almost 60% of the entire harbour area and the flushing times of these segments are low (explaining why the graph in Fig. 11a departs from an exponential function in the first 20 days), this confirms our conclusion that tidal exchange is rapid in the vicinity of the entrance.

Discussion and Conclusion

The goals of this study are to understand the tide-induced residual circulation and dispersion in the Sydney Harbour, the exchange of water between harbour and the open ocean and the relative flushing capacity of different parts of the harbour.

For this study the harbour has been assumed to be of uniform density which implies that there is no information
regarding vertical circulation or changes in the horizontal circulation due to lateral density gradients. Since depth-averaged tidal models cannot represent transport processes that depend on the vertical structure of the currents, the estimates of dispersion and exchange calculated here must be regarded as at the lower end of the range.

Sydney Harbour has a complex bathymetry which is reflected in the need for a small amount (0–2 m² s⁻¹) of horizontal diffusion in spite of the use of fine resolution (150 m). The Eulerian residual field of Sydney Harbour consists of a series of horizontal gyres. The results show that inclusion of the nonlinear advective terms in the momentum equations mainly produces residual currents in the harbour. Therefore it is the interaction of the tidal current with the complicated geometry of the harbour that is producing these gyres. Inclusion of homogeneous wind stress in the model shows that wind stress in the harbour that is producing these gyres. The results show that inclusion of the nonlinear advective terms in the momentum equations mainly produces residual currents in the harbour. Therefore it is the interaction of the tidal current with the complicated geometry of the harbour that is producing these gyres. Inclusion of homogeneous wind stress in the model shows that wind stress does not significantly change the pattern of the residual circulation. Some additional gyres are formed and their sense of rotation depends on the wind direction.

Inclusion of the $S_2$ tide in the input forcing does not significantly change the Eulerian residual velocity estimated with the $M_2$ tide alone; hence there is no significant nonlinear interaction between the $M_2$ and $S_2$ tides in the harbour region, and residuals are produced mainly by the $M_2$ tide.

Though the Eulerian residual field consists of horizontal gyres the dispersion processes depend on both the tidal field and the residual flow field. The Lagrangian residual field depends on the phase of the tide and does not show any resemblance to the Eulerian residual field. This suggests that even if the Eulerian residual circulation is known in great detail, the net transport of material over a tidal cycle can not be determined from this knowledge. Therefore, in the case of Sydney Harbour, the Lagrangian residual velocity should be regarded as a map that indicates the net displacement of parcels released at a particular phase of the tide.

Tracer simulations show that tidal exchanges are rapid at the beginning and tidal mixing is limited in the vicinity of the entrance. The 225-day flushing time of the whole harbour is the top of the range of flushing times, since only the tidal mixing processes are modelled. Simulations also show that when materials are introduced all over the harbour Segment 2 takes the longest to flush.

The flushing rates of different segments of the harbour vary significantly. The degree of flushing decreases upstream. Since the harbour does not in practice become homogenized by tidal mixing, the region near the harbour mouth would be flushed much more rapidly than other regions. Since the path travelled by the particles depends on the tidal phase, the flushing time would depend on the time of release.

This study shows that, in the case of Sydney Harbour, knowledge of both Eulerian and Lagrangian residual circulation is useful. While Lagrangian residual circulation determines the net transport of water-borne materials, Eulerian residual circulation helps to correctly interpret the measurements from the current meters.

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